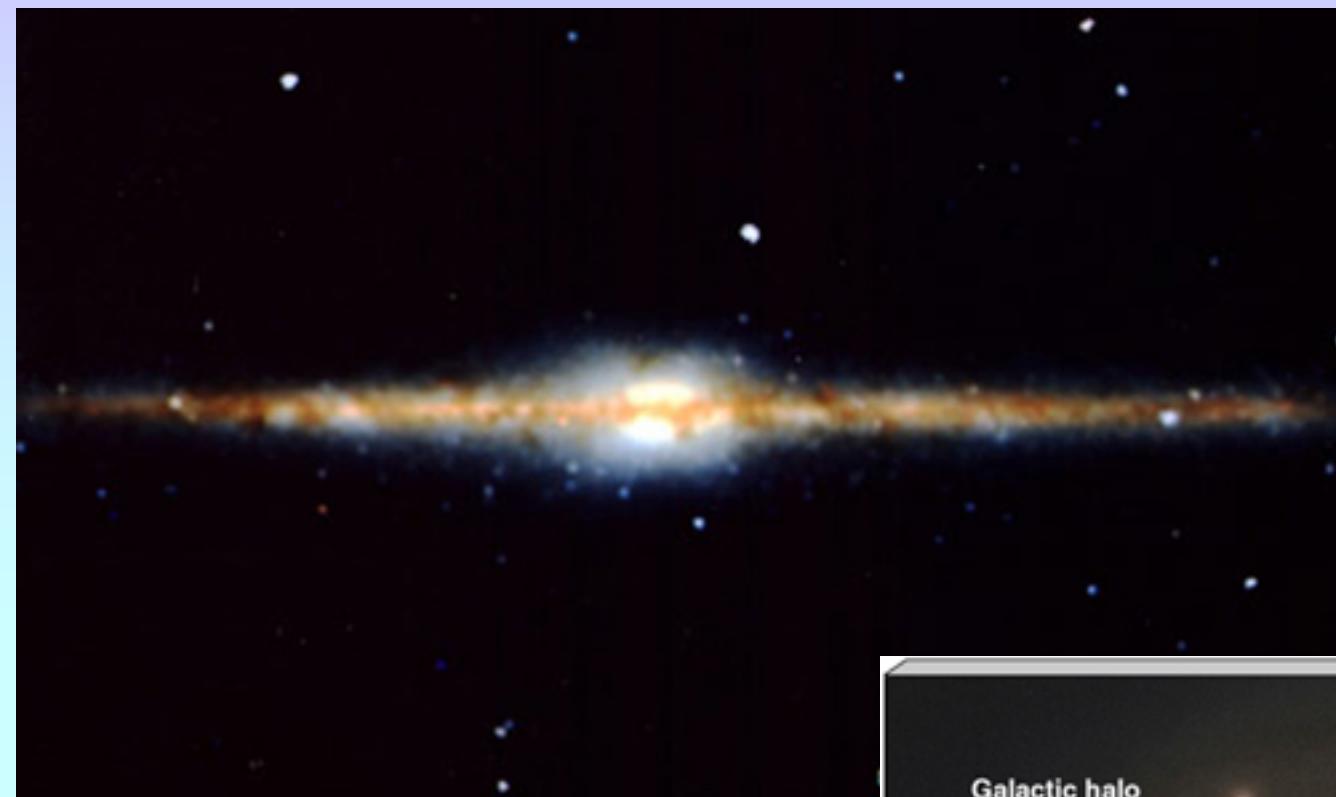
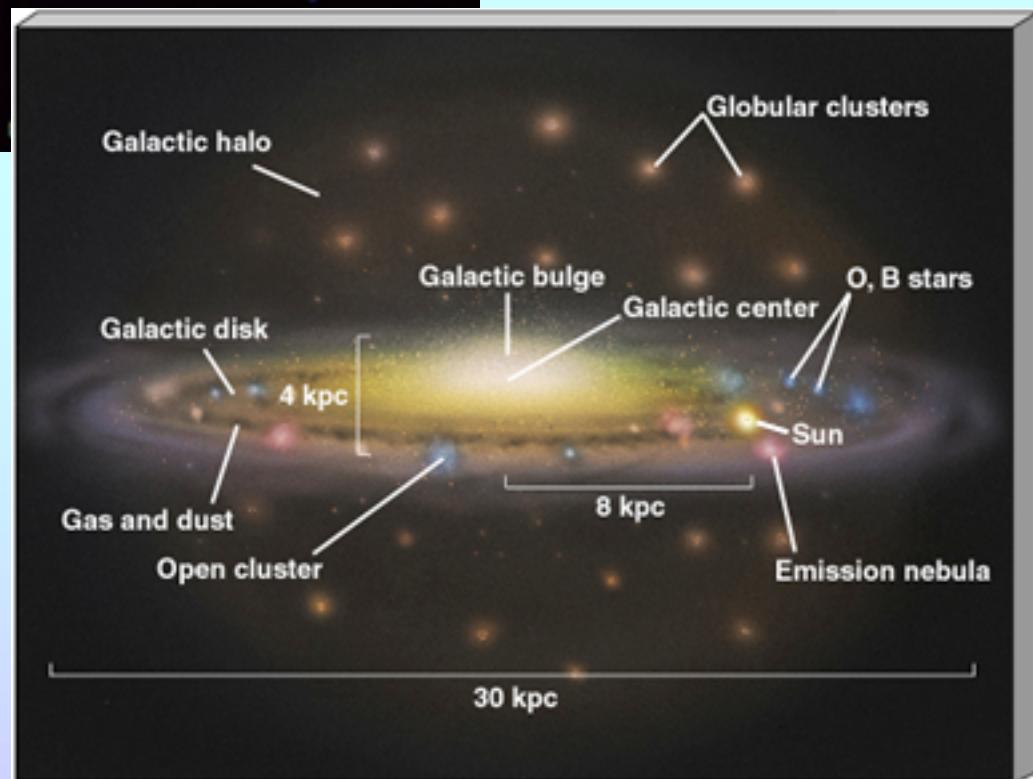
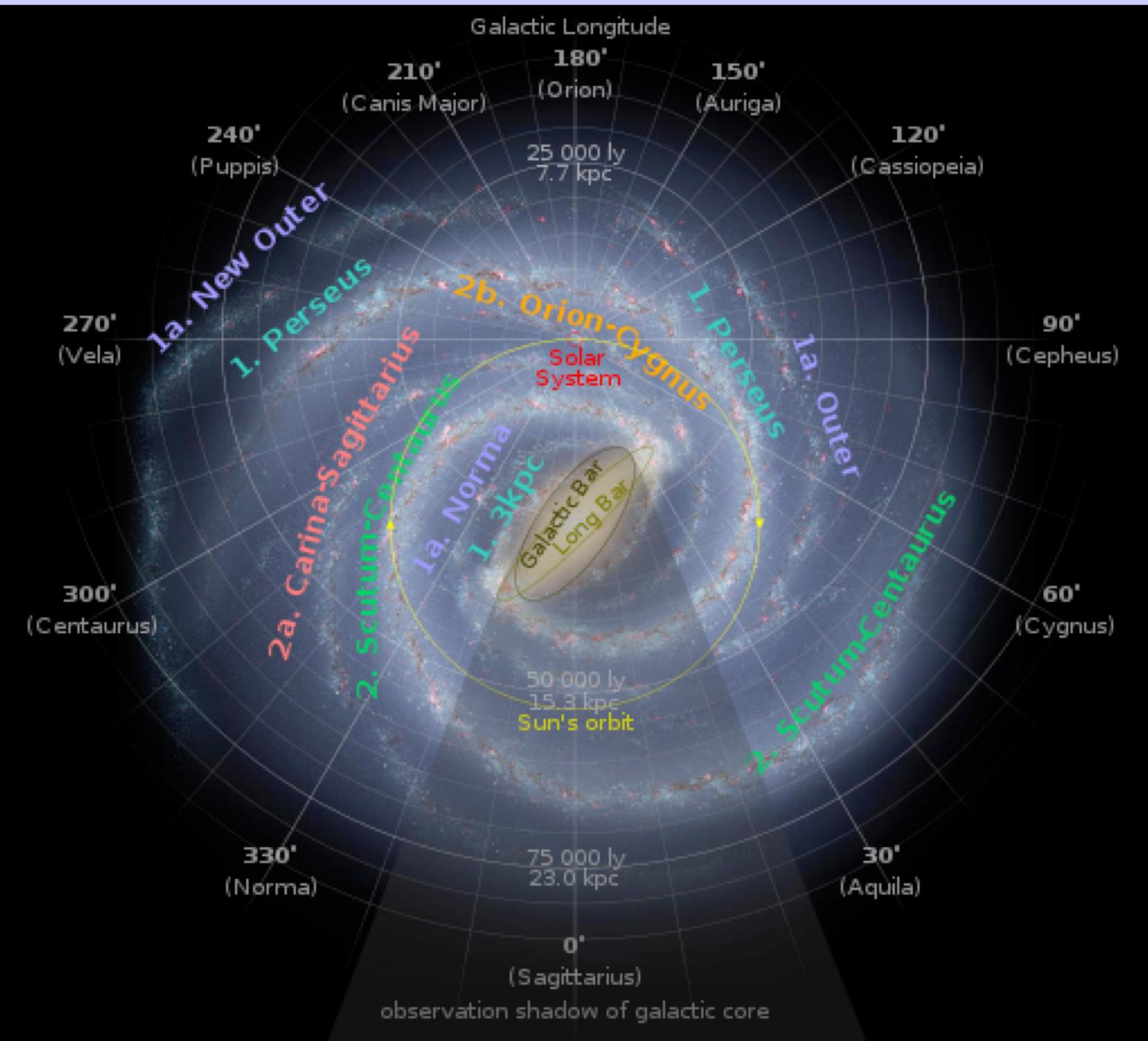


Far-IR all-sky  
image of the  
Milky Way



Schematic of the Galaxy





# Parameterization

The Milky Way (and other spiral galaxies) are usually described via several components:

- Thin Disk: the area where most of the cold ISM and young stars reside. The scale height of this area is  $\sim 300$  pc. The Sun is about  $\sim 25$  pc above the Galactic plane.
- Thick Disk: a “puffed up” disk with scale height  $\sim 1$  kpc. In the solar neighborhood, this component is  $\sim 10\%$  that of the thin disk.
- Bar: an intermediate age ( $\sim 3$  Gyr) bar, oriented close to the line of sight.
- Halo: A slightly flattened ( $b/a \sim 0.6$ ) distribution of old stars extending  $\sim 40$  kpc. In the solar neighborhood, the halo density is  $\sim 0.5\%$  of the thin disk.
- Dark Halo: A dark (roughly spherical?) halo extending from  $\sim 50$  to  $\sim 100$  kpc.

# The Rotation Curve

Most H I is rotating about the center of the Galaxy. At any Galactic longitude,  $l$ , we should see gas with radial velocities

$$V_r = r\omega \cos \theta - R_\odot \omega_\odot \sin l$$

From the law of sines

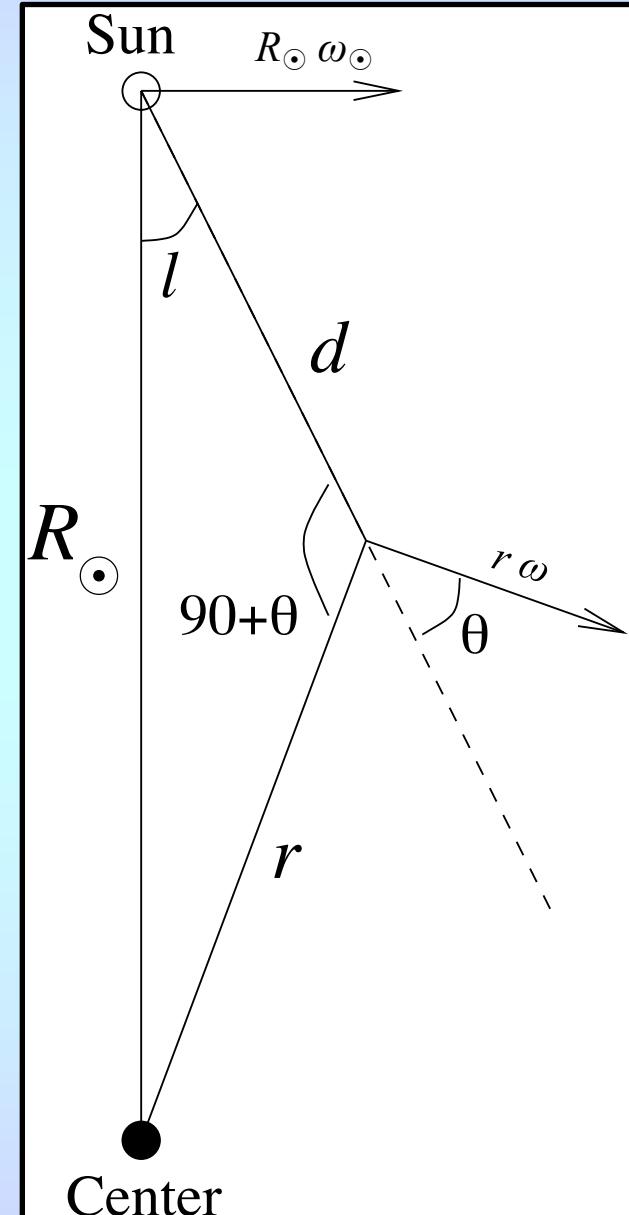
$$\frac{\sin l}{r} = \frac{\sin(90 + \theta)}{R_\odot} = \frac{\cos \theta}{R_\odot}$$

So

$$V_r = R_\odot \omega \sin l - R_\odot \omega_\odot \sin l = R_\odot (\omega - \omega_\odot) \sin l$$

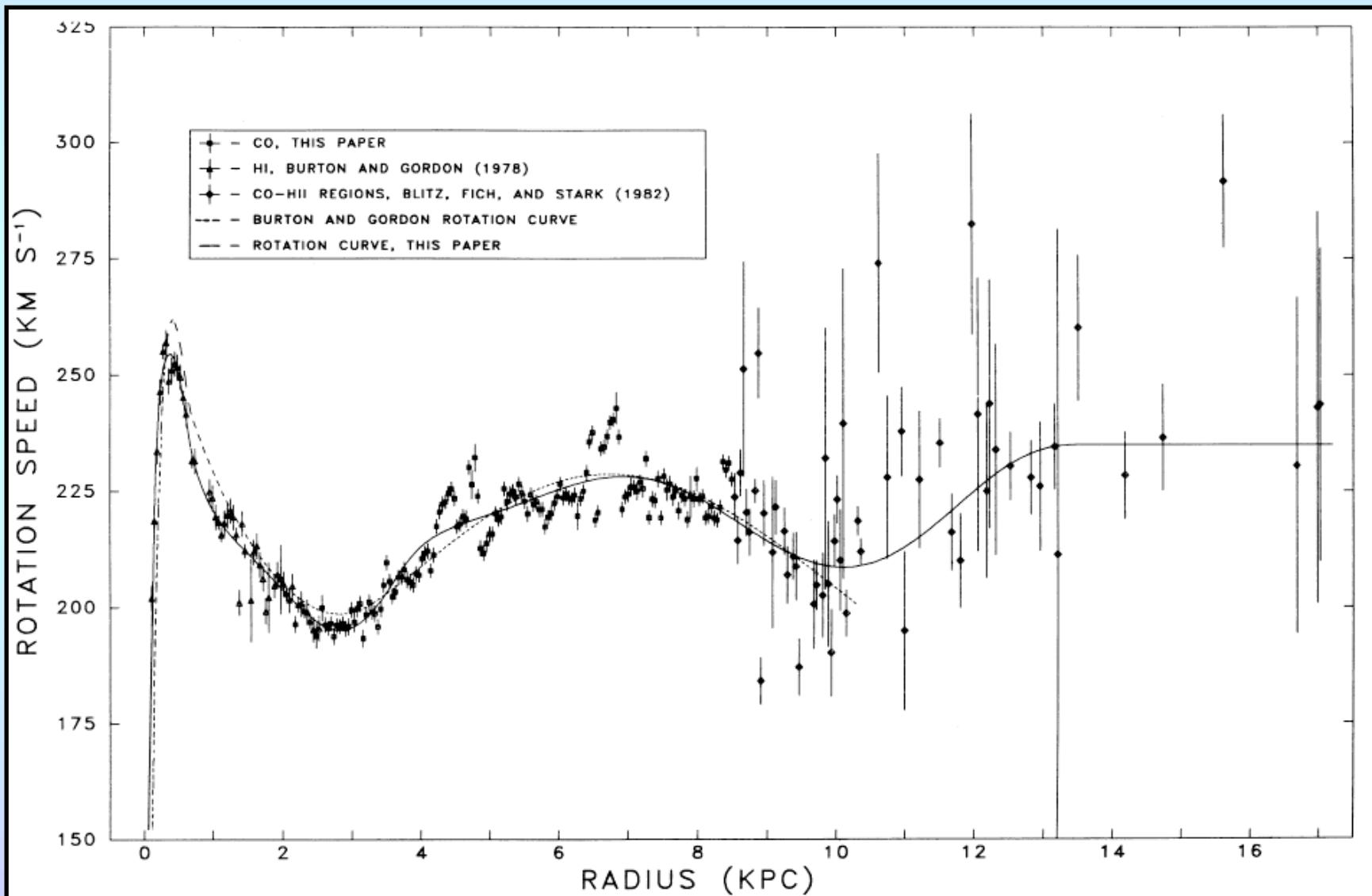
Since the maximum velocity is obtained when the gas motion is along the line-of-sight, i.e., at  $r_T = R_\odot \sin l$ , we can use this to derive the Milky Way's rotation curve,  $\omega(r_T)$

$$V_{\max} = R_\odot [\omega(r_T) - \omega_\odot] \sin l$$



# Milky Way Rotation Curve

At  $R < 3$  kpc, the derived curve is strongly affected by the breakdown of the circular motion hypothesis due to the central bar.



# The Local Shear and Vorticity: Oort Constants

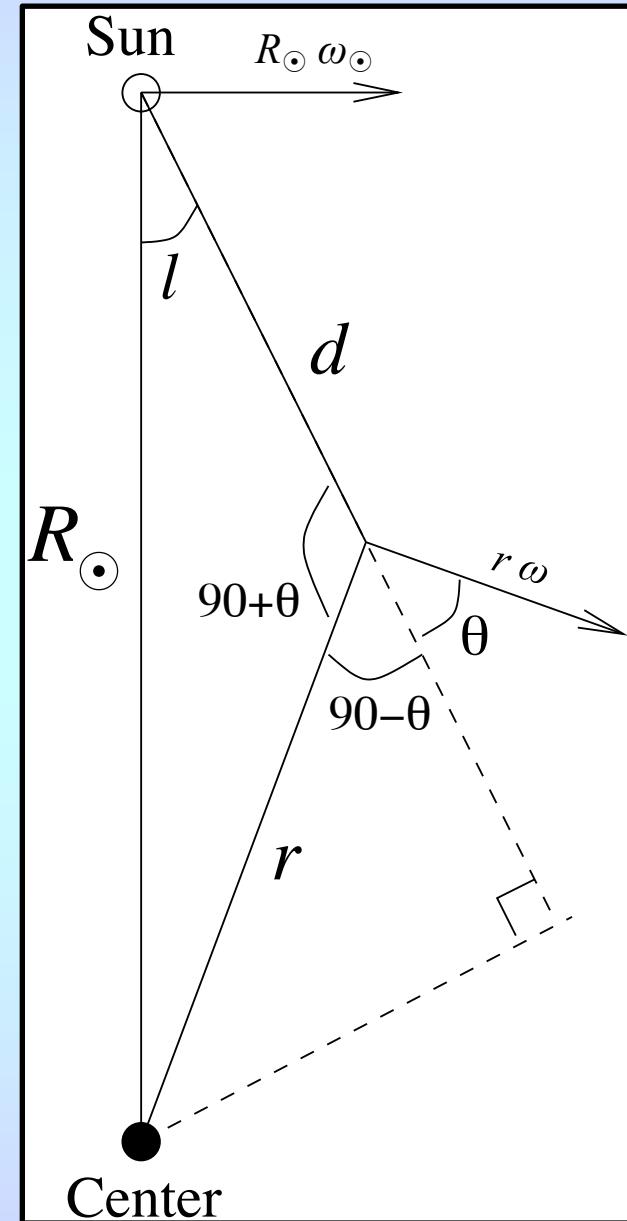
The traditional method for describing the *local* behavior of the Galactic disk is through Oort Constants. We begin with the observed radial velocity

$$V_R = R_\odot (\omega - \omega_\odot) \sin l$$

If  $\Theta(r) = r \omega$  is the circular velocity at  $r$ , then to first order

$$\begin{aligned} \omega - \omega_\odot &= \left( \frac{d\omega}{dr} \right)_{R_\odot} (r - R_\odot) \\ &= \left\{ \frac{1}{R_\odot} \left( \frac{d\Theta}{dr} \right)_{R_\odot} - \frac{\Theta_\odot}{R_\odot^2} \right\} (r - R_\odot) \end{aligned}$$

But for small distances,  $R_\odot - r \sim d \cos l$ , so



# The Local Shear and Vorticity: Oort Constants

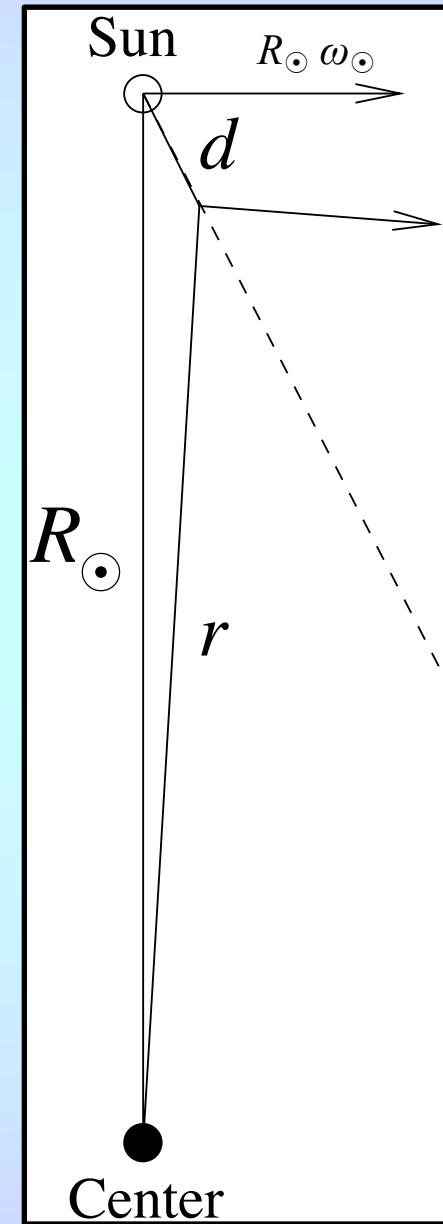
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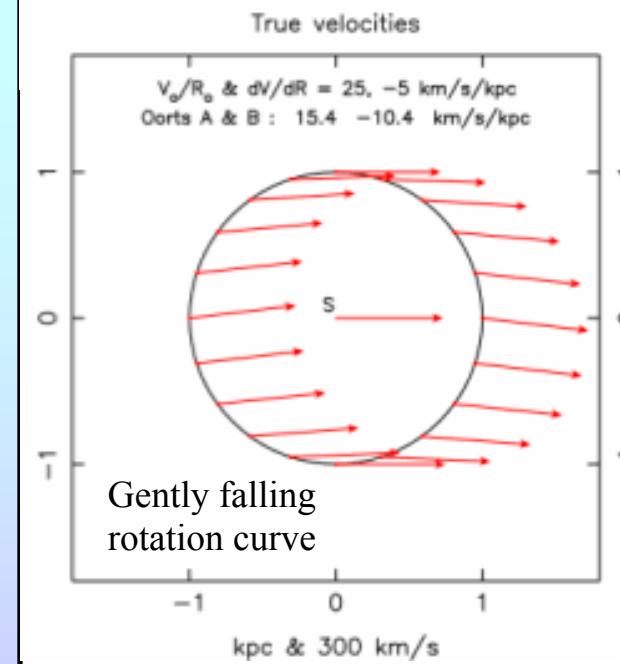
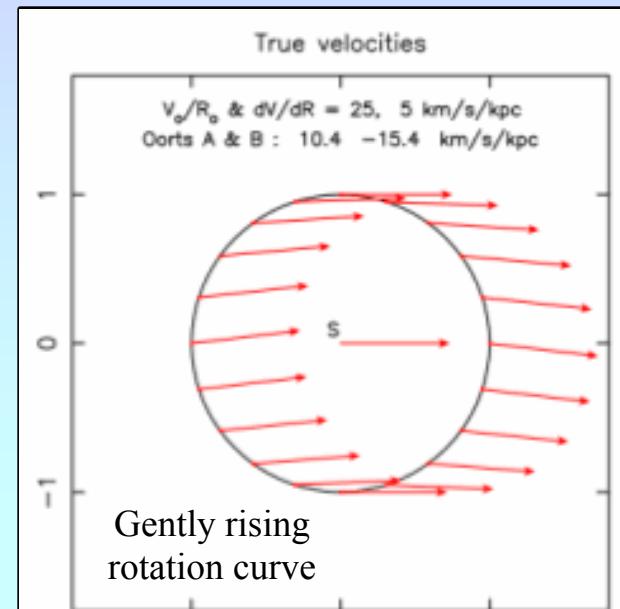
But for small distances,  $R_\odot - r \sim d \cos l$ , so



# The Local Shear and Vorticity: Oort Constants

$$\begin{aligned}
 V_R &= R_\odot \left\{ \frac{1}{R_\odot} \left( \frac{d\Theta}{dr} \right)_{R_\odot} - \frac{\Theta_\odot}{R_\odot^2} \right\} (r - R_\odot) \sin l \\
 &\approx \left\{ \frac{\Theta_\odot}{R_\odot} - \left( \frac{d\Theta}{dr} \right)_{R_\odot} \right\} d \sin l \cos l \\
 &\approx \frac{1}{2} \left\{ \frac{\Theta_\odot}{R_\odot} - \left( \frac{d\Theta}{dr} \right)_{R_\odot} \right\} d \sin 2l \\
 &\approx A d \sin 2l
 \end{aligned}$$

The Oort Constant  $A \sim 13$  km/s/kpc describes the shear in the local velocity field.



# The Local Shear and Vorticity: Oort Constants

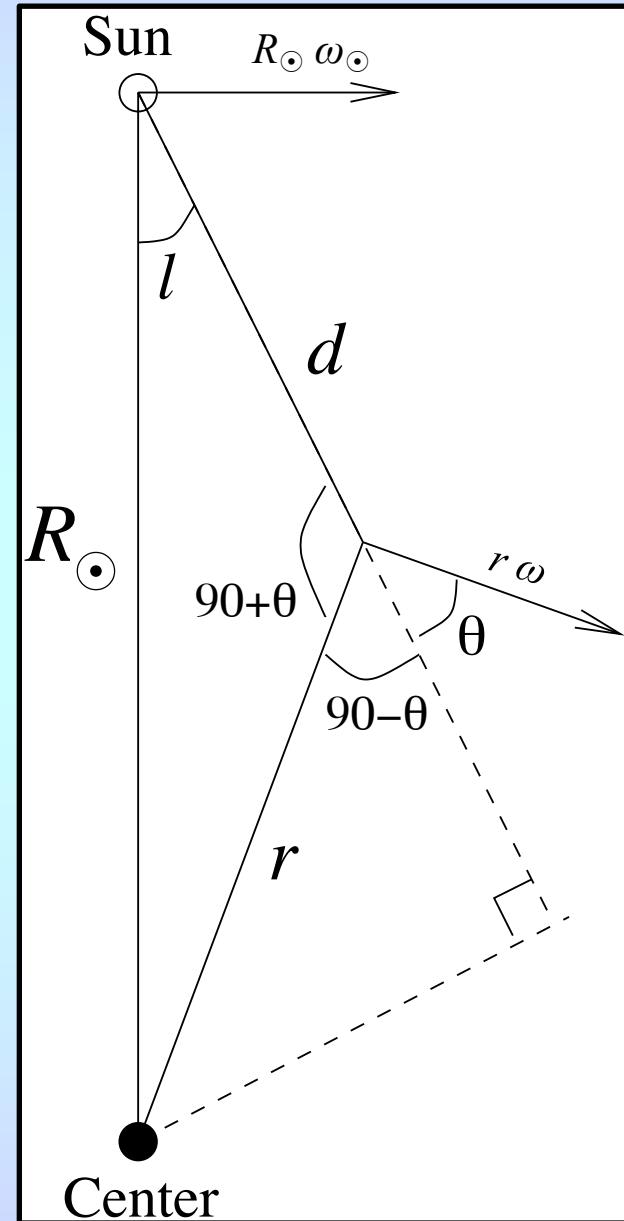
Alternatively, if we consider the transverse (i.e., the proper) motion of nearby objects

$$\begin{aligned}V_T &= r\omega \sin \theta - R_\odot \omega_\odot \cos l \\&= \omega(R_\odot \cos l - d) - R_\odot \omega_\odot \cos l \\&= R_\odot(\omega - \omega_\odot) \cos l - \omega d\end{aligned}$$

and expand  $\omega d$  so that

$$\begin{aligned}\omega d &\approx d \left[ \omega_\odot + \left( \frac{d\omega}{dr} \right)_{R_\odot} (r - R_\odot) + \dots \right] \\&\approx \omega_\odot d - \left( \frac{d\omega}{dr} \right)_{R_\odot} d^2 \cos l\end{aligned}$$

If we now substitute this in, along with our previous expression for  $\omega - \omega_\odot$ , then



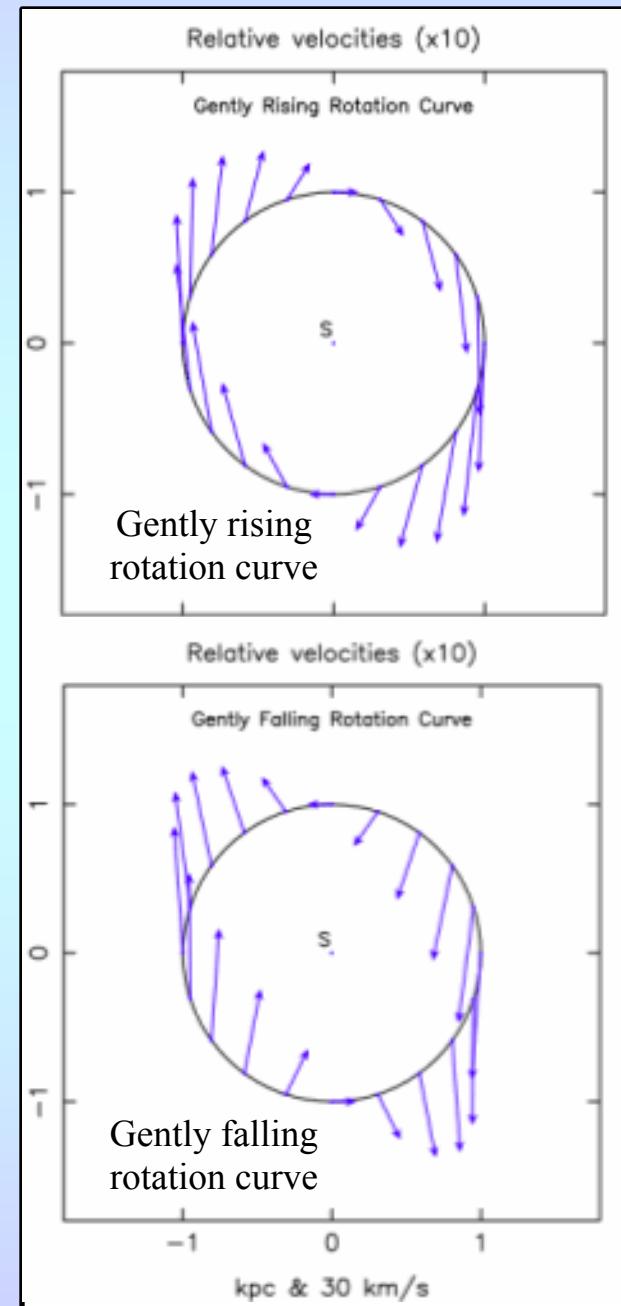
# The Local Shear and Vorticity: Oort Constants

$$\begin{aligned}
 V_T &= \left[ \left( \frac{d\Theta}{dr} \right)_{R_\odot} - \frac{\Theta_\odot}{R_\odot} \right] (r - R_\odot) \cos l - \omega_\odot d \\
 &= \left[ \frac{\Theta_\odot}{R_\odot} - \left( \frac{d\Theta}{dr} \right)_{R_\odot} \right] d \cos^2 l - \frac{\Theta_\odot}{R_\odot} d
 \end{aligned}$$

Finally, if we use  $\cos^2 l = \frac{1}{2}(1+\cos 2l)$ , and note that the proper motion,  $\mu = V_T / 4.74 d$ ,

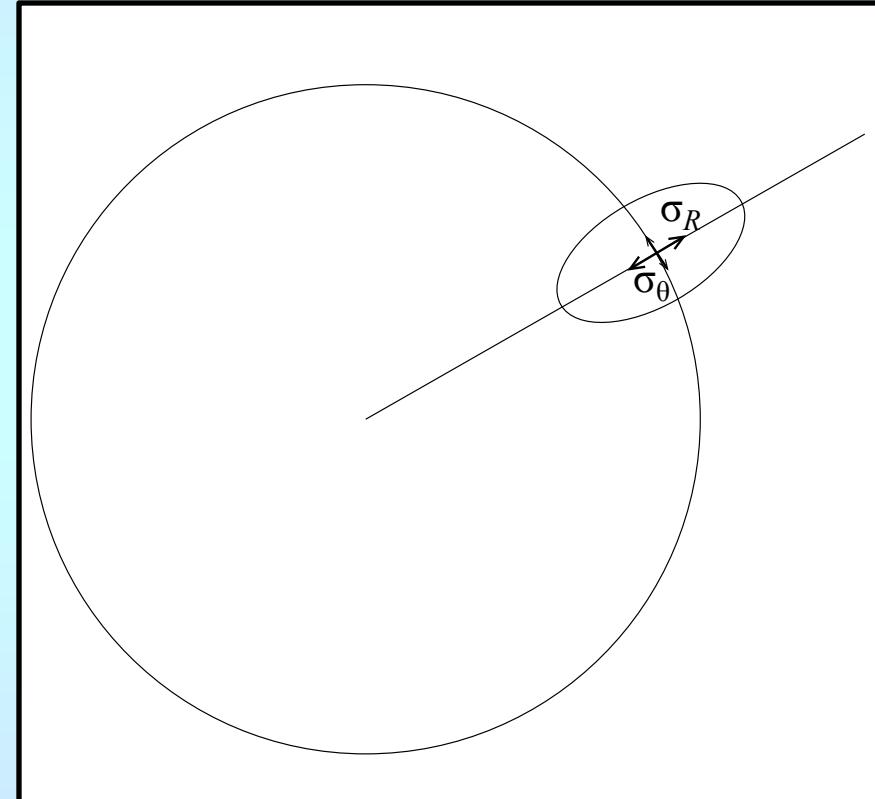
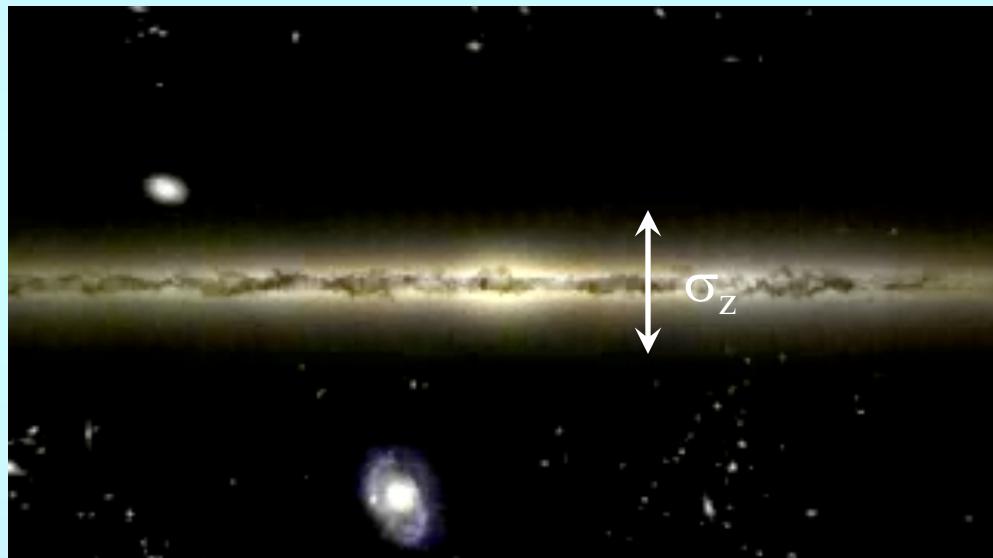
$$\begin{aligned}
 V_T &= \frac{1}{2} \left[ \frac{\Theta_\odot}{R_\odot} - \left( \frac{d\Theta}{dr} \right)_{R_\odot} \right] d \cos 2l - \frac{1}{2} \left[ \frac{\Theta_\odot}{R_\odot} + \left( \frac{d\Theta}{dr} \right)_{R_\odot} \right] d \\
 &= d(A \cos 2l + B) \quad \text{and} \quad \mu = \frac{A \cos 2l + B}{4.74}
 \end{aligned}$$

The coefficient  $B = -10.2 \text{ km/s/kpc}$  describes the local rotation of the solar neighborhood; it can be measured from proper motion data without any knowledge of distance.



# Velocity Ellipsoid

The stars in the solar neighborhood are not orbiting together. While  $V_{\text{rot}}$  describes their bulk motion, their residuals are described by a velocity ellipsoid,  $\sigma_R$ ,  $\sigma_\theta$ , and  $\sigma_z$ .



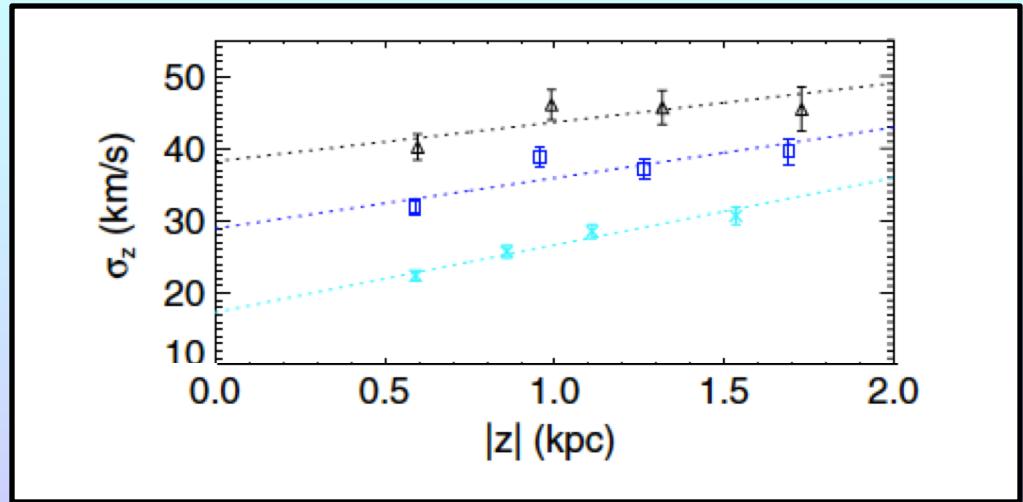
The values  $\sigma_R$  and  $\sigma_\theta$  probe the disk scattering and place constraints on stability. The vertical velocity dispersion,  $\sigma_z$  is more useful, as it allows us to measure disk mass.

# Disk Thickness

The thickness of a disk galaxy is usually parameterized via an exponential,  $N(z) \propto \exp\{-z/z_0\}$ .

Object	Age	Scale Height	$\sigma_z$
O stars	$\sim 3 \times 10^6$	$\sim 50$ pc	$\sim 5$ km/s
B stars	$\sim 1.5 \times 10^7$	$\sim 60$ pc	$\sim 8$ km/s
A stars	$\sim 5 \times 10^8$	$\sim 120$ pc	$\sim 9$ km/s
F stars	$\sim 3 \times 10^9$	$\sim 190$ pc	$\sim 11$ km/s
G stars	$\sim 10 \times 10^9$	$\sim 340$ pc	$\sim 15$ km/s
K stars	$\sim 15 \times 10^9$	$\sim 350$ pc	$\sim 17$ km/s

The  $z$ -velocity dispersion is almost independent of height above the plane.



# One Slide on Stellar Dynamics

The stellar distribution in the  $z$ -direction is related to  $\sigma_z$  through the Boltzmann and Poisson's equation. Stars are essentially collisionless particles moving through a 6-dimensional phase space ( $x, y, z, v_x, v_y, v_z$ ). If  $\omega$  is an object's 6-D coordinate, and  $f(\omega)$  the distribution of stars, then by analogy to Euler's equation for fluids

$$\frac{\partial f}{\partial t} + \nabla \cdot (f \dot{\omega}) = 0$$

The collisionless Boltzmann equation is rarely used. Instead, one typically integrates over various quantities to derive a series of Jean's equations, which can then be combined with Poisson's equation to yield constraints on the mass distribution.

# $\sigma_z$ and Disk Mass

For thin disks, the Jeans equation in cylindrical coordinates can be written as

$$\frac{\partial}{\partial z} \left\{ \frac{1}{v} \frac{\partial}{\partial z} (v \sigma_z^2) \right\} = -4\pi G \rho$$

where  $v$  is the stellar density and  $\rho$  the mass density. This equation can be solved to yield the  $v$  as a function of  $z$  and  $\sigma_z(z)$ . Note that if  $\sigma_z$  is independent of  $z$  (an isothermal disk), this further simplifies to

$$\frac{\partial}{\partial z} \left\{ \frac{1}{v(z)} \frac{\partial v(z)}{\partial z} \right\} = -\frac{4\pi m G v(z)}{\sigma_z^2}$$

where  $m$  is the stellar mass. The solution to this equation is

$$\rho(z) = \rho_0 \operatorname{sech}^2 \left( \frac{z}{2z_0} \right) \quad \text{where} \quad z_0 = \left( \frac{\sigma_z^2}{8\pi G \rho_0} \right)^{1/2}$$

Note that this leads to a simple relation between velocity dispersion and  $\Sigma$ , the surface mass of the disk:

$$\sigma_z^2 = 2\pi G \Sigma z_0$$

# $\sigma_z$ and Disk Mass

Note: a  $\text{sech}^2$  law is not that different from an exponential disk.

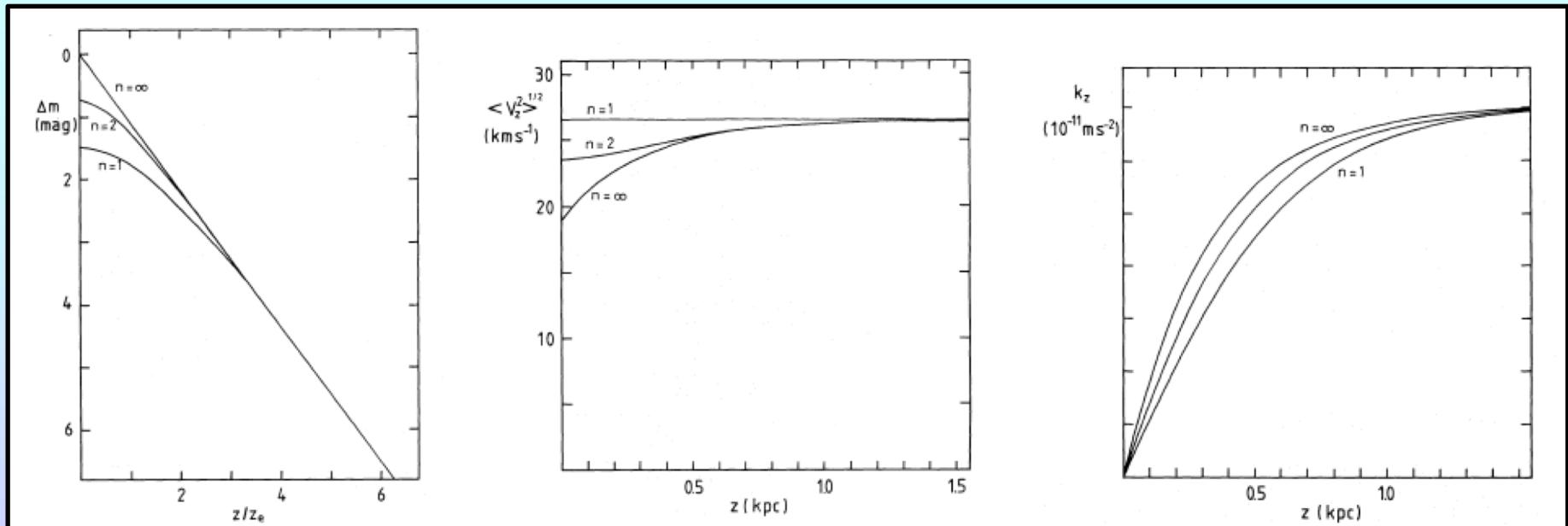
$$\text{sech}(z) = \frac{1}{\cosh(z)} = \frac{2}{e^z + e^{-z}} \approx e^{-z}$$

In fact, there is a family of solutions for spiral disks. In general,

$$N(z) \propto \text{sech}^{2/n} \left( \frac{n z}{2 z_0} \right) \quad \text{where}$$

$n = 1$  implies an isothermal disk

$n = \infty$  implies an exponential disk

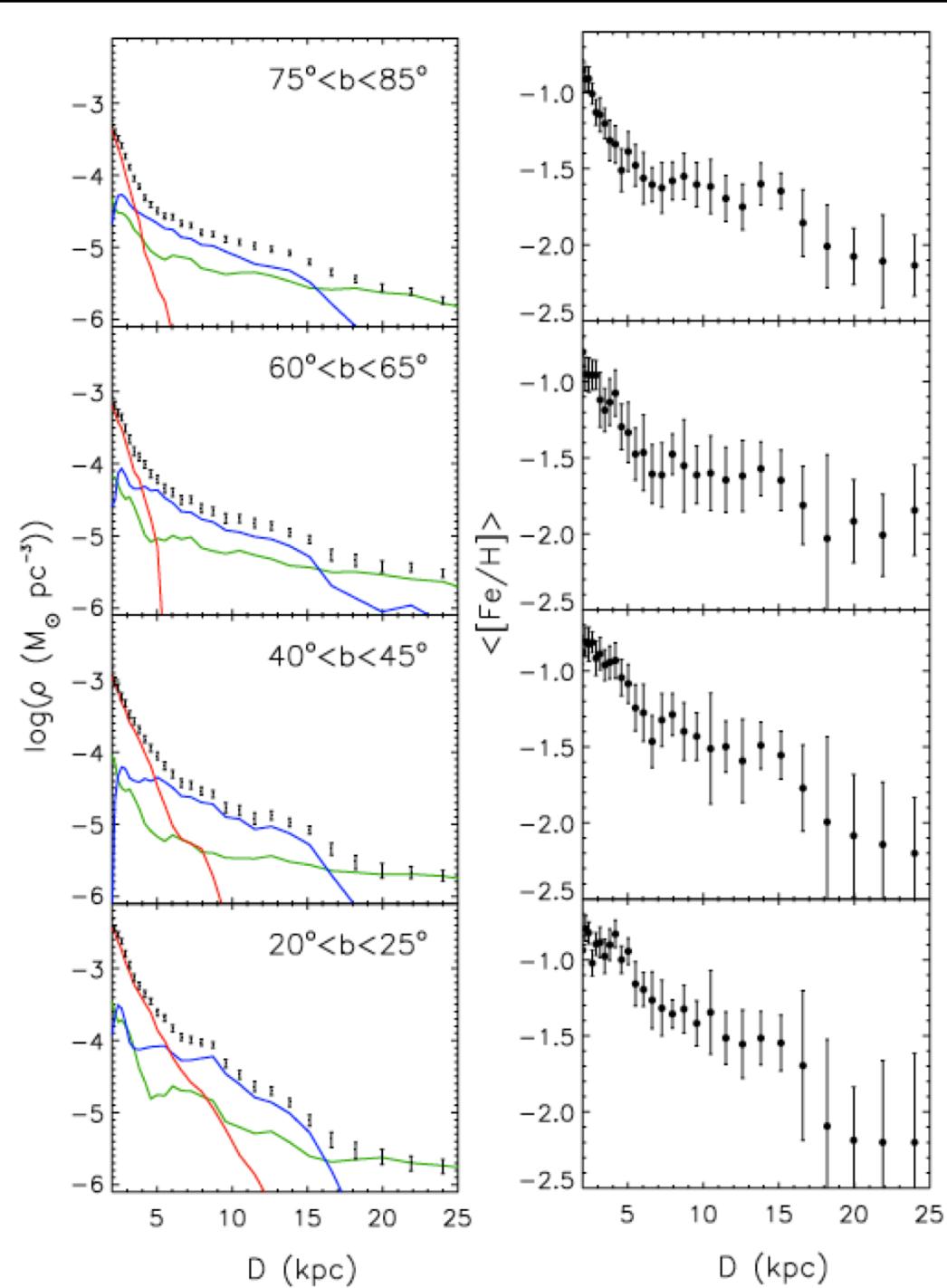


# The Radial Distribution

Once outside the bar, the disk's surface brightness can best be fit with a simple exponential,

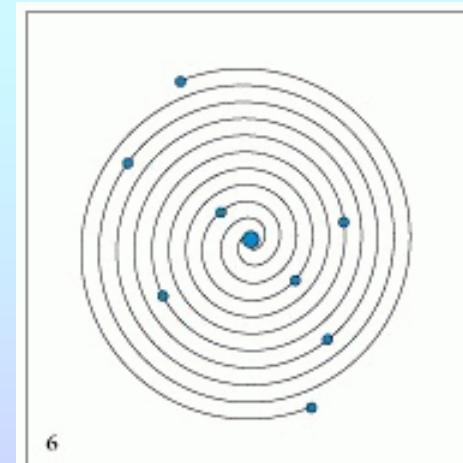
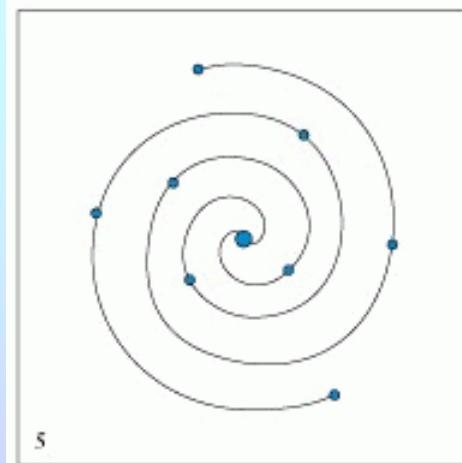
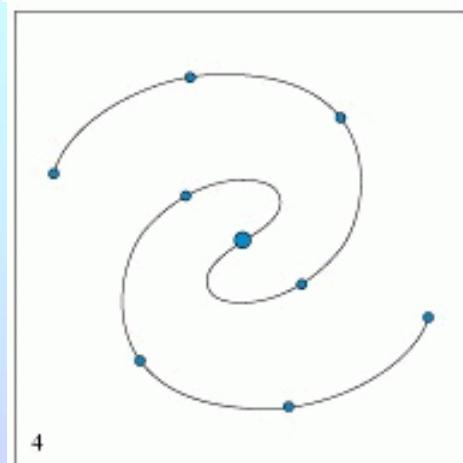
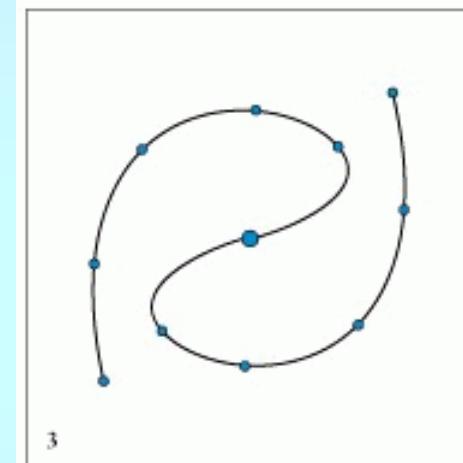
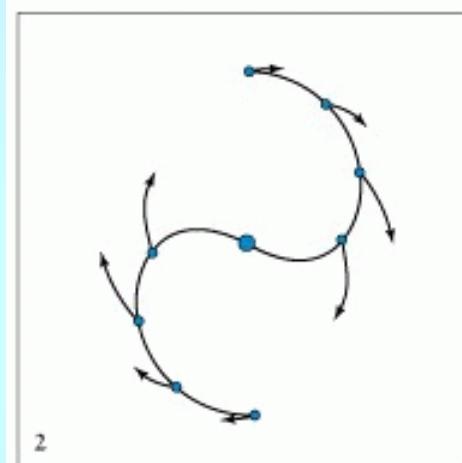
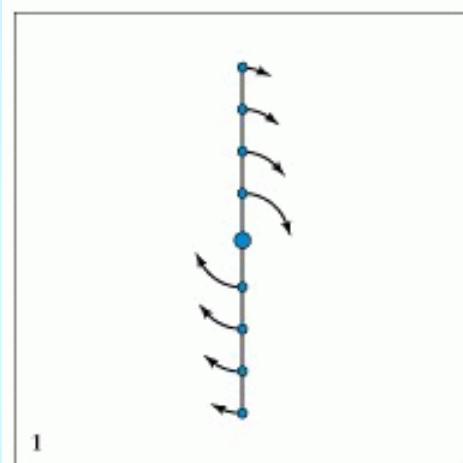
$$I(r) = I_0 \exp\left(-\frac{r}{r_0}\right)$$

The scale length,  $r_0$ , of the disk is somewhat controversial, but is likely  $\sim 3$  kpc (for both the thin and thick components). The metallicity of the disk also declines with a somewhat similar scale length (though the  $\alpha$ -elements may scale differently than the Fe-peak elements).

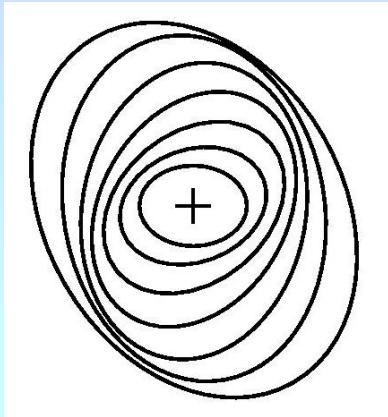


# Spiral Arms

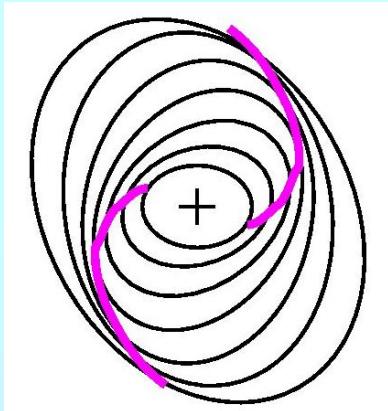
Because orbital frequency depends on distance, an initial line of stars will quickly be drawn out into a spiral, and wrapped up in just a few orbits. Arms like this can't last more than about  $\sim 1$  Gyr.



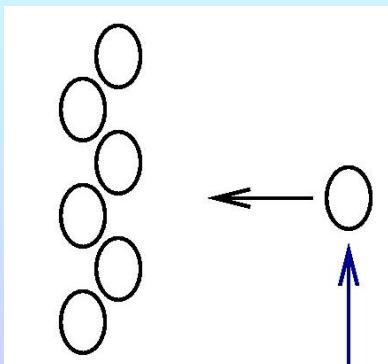
# Spiral Density Waves



- The orbits in spiral galaxies are not quite circles – they are ellipses. These ellipses are slightly tilted with respect to each other.

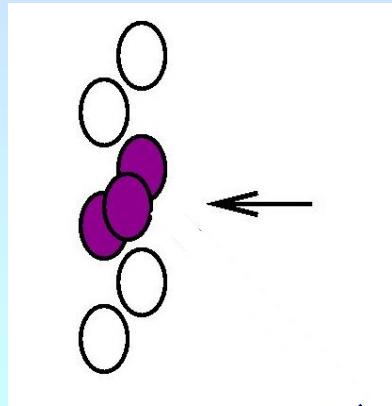


- Thus there are regions of slightly higher density than their surroundings. The higher density means higher gravity.

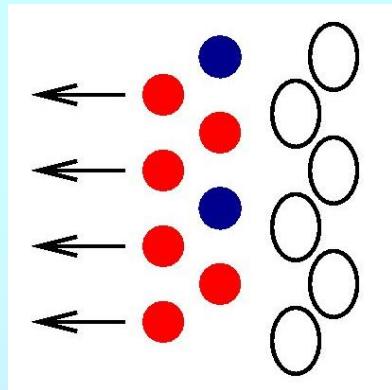


- An object (such as a gas cloud) will be attracted to these regions and will drift towards them.

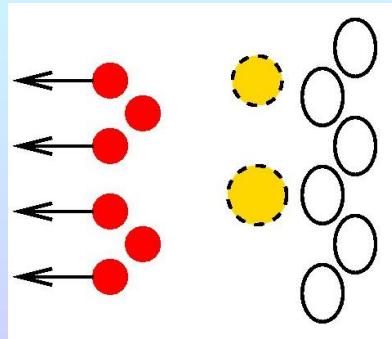
# Spiral Density Waves



- When the gas cloud collides with other gas clouds, stars will be formed. (This is where most of the galaxy's star formation takes place.)



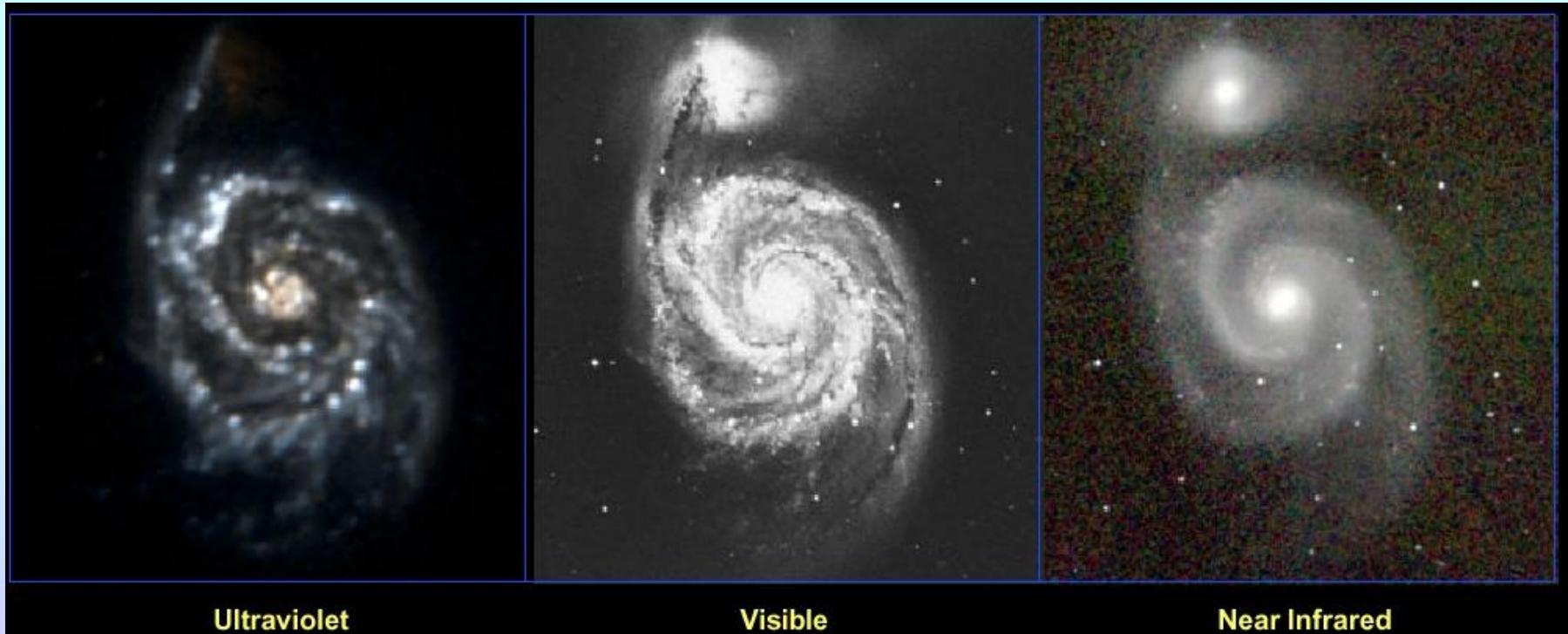
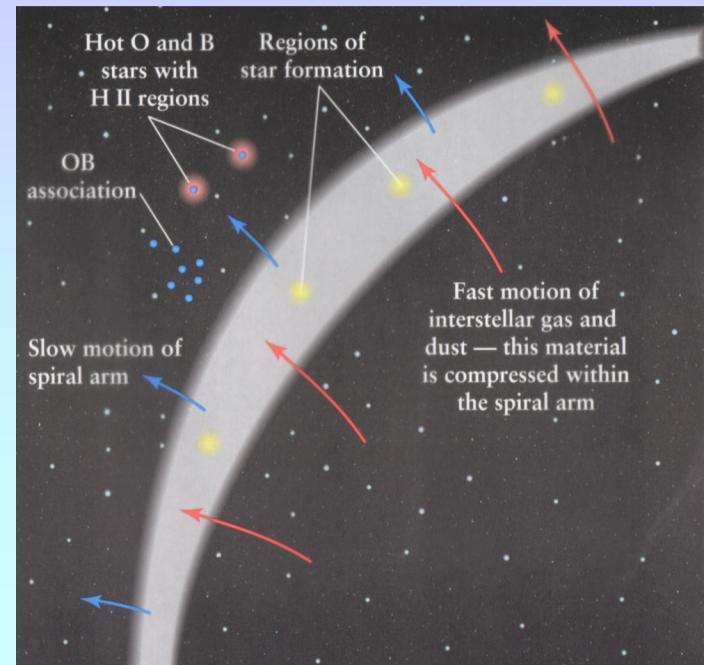
- Star will form over the whole range of stellar masses. A typical Initial Mass Function (IMF) is  $N(M) \propto M^s$ , with  $s = -2.35$  (the Salpeter mass function). More modern IMFs do exist.



- High-mass (O and B main sequence) stars don't go far before they go supernova. The brightest (and bluest) of a galaxy's stars will never be far from the spiral arm.

# Spiral Density Waves

Since all the bright blue stars die before leaving the spiral arm, the **spiral density waves** must show up better at ultraviolet wavelengths. In general, the density enhancement in the spiral arms is just a few percent.

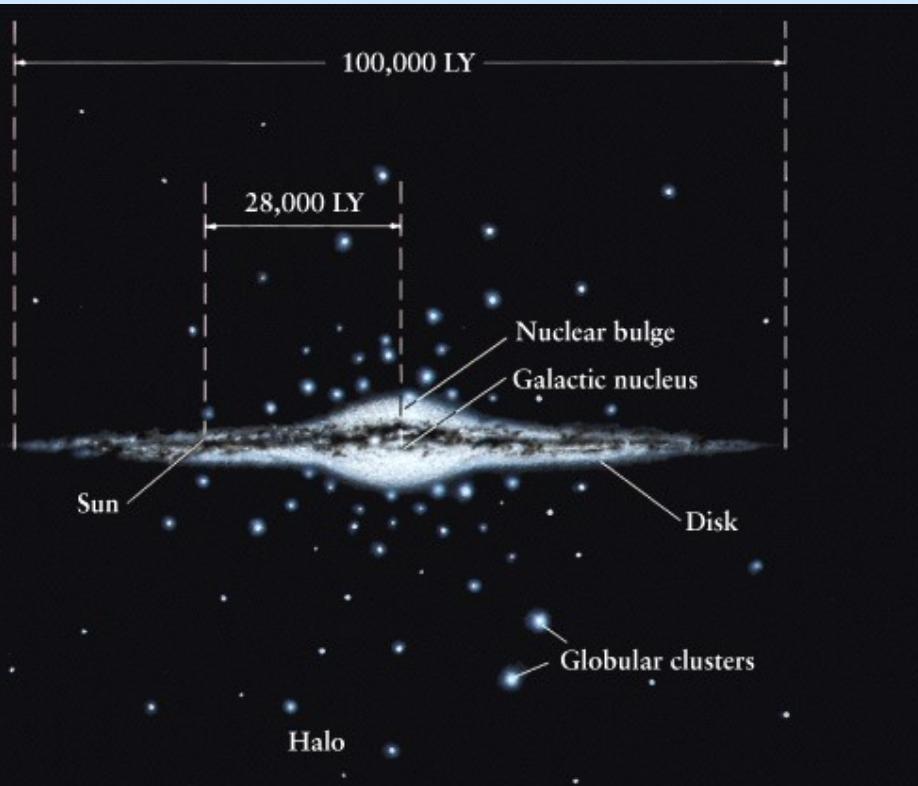


# The Stellar Halo

The spheroidal component of the galaxy declines roughly as  $\rho \propto r^{-n}$ , with  $n \sim 2.77$ . Traditionally, this halo was thought to formed early on, before the collapse of the disk (Eggen, Lynden-Bell, & Sandage 1962). However, it is more likely that this component has been built up hierarchically through collisions with small galaxies. This component extends at least  $\sim 50$  kpc.

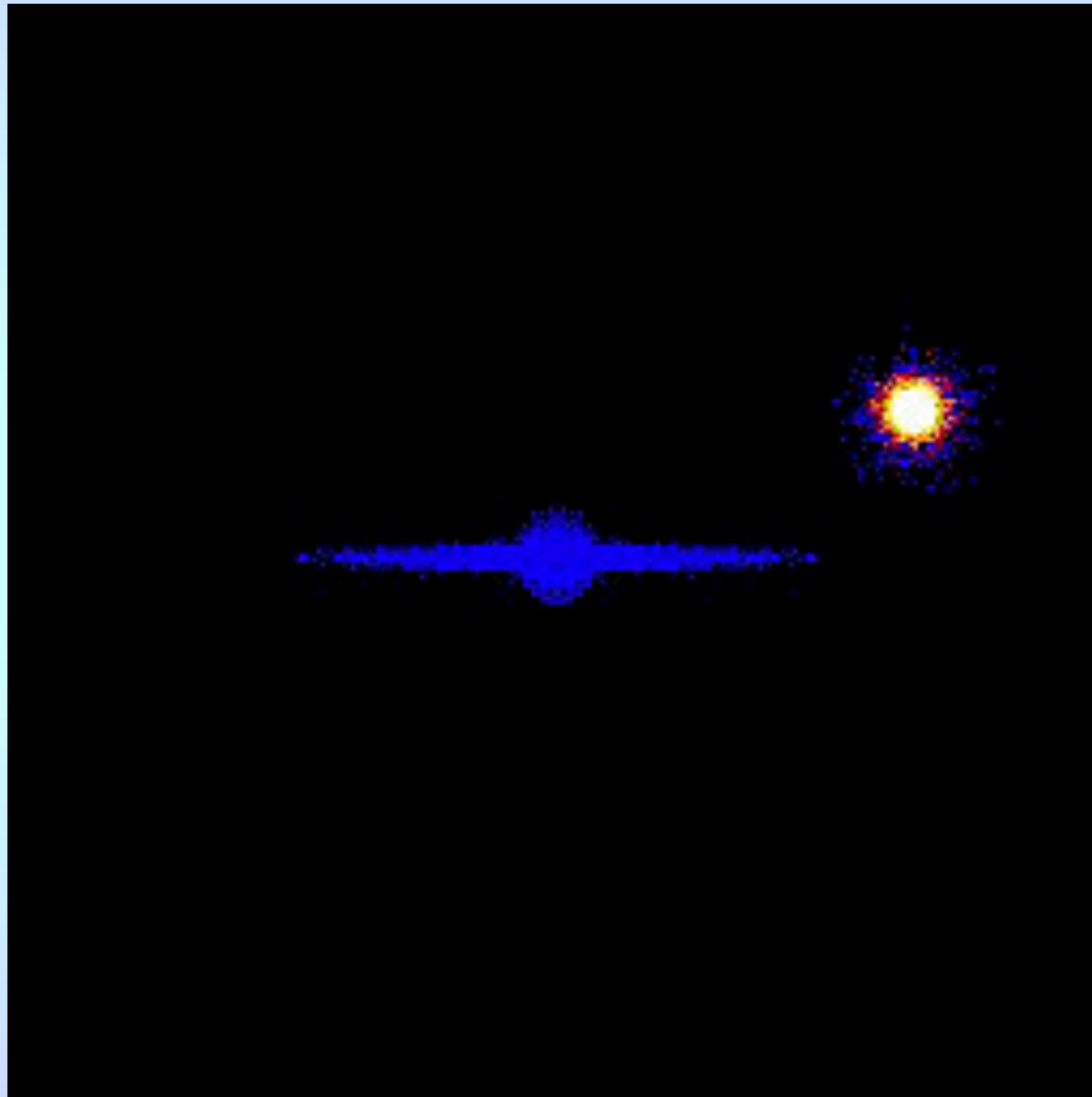
Halo stars are generally metal poor, and do not rotate with the disk. They are sometimes referred to as “high velocity stars”, since their motions deviate substantially from the local standard of rest.

# Milky Way Cannibalism

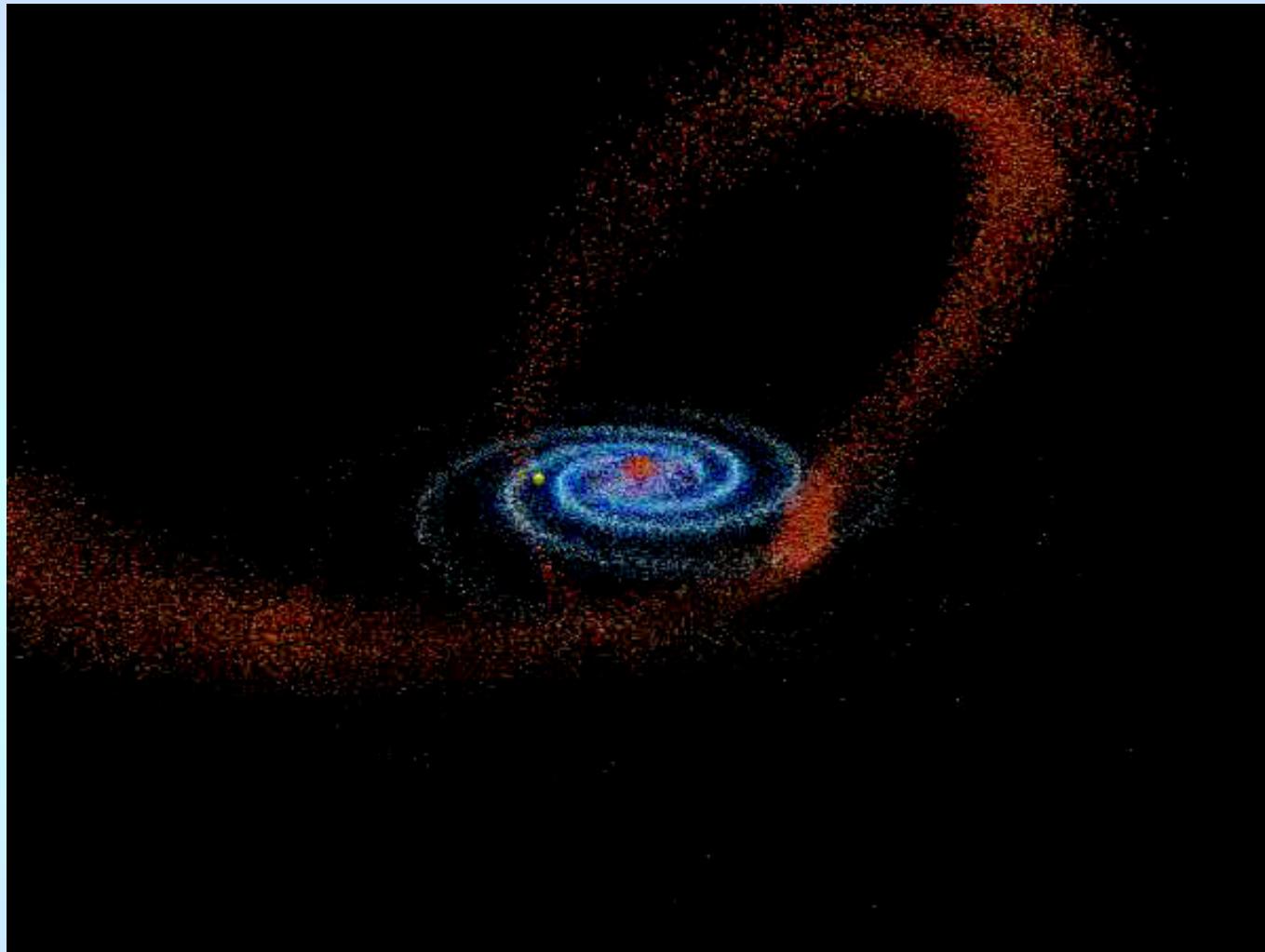


←A small dwarf galaxy

# Milky Way Cannibalism



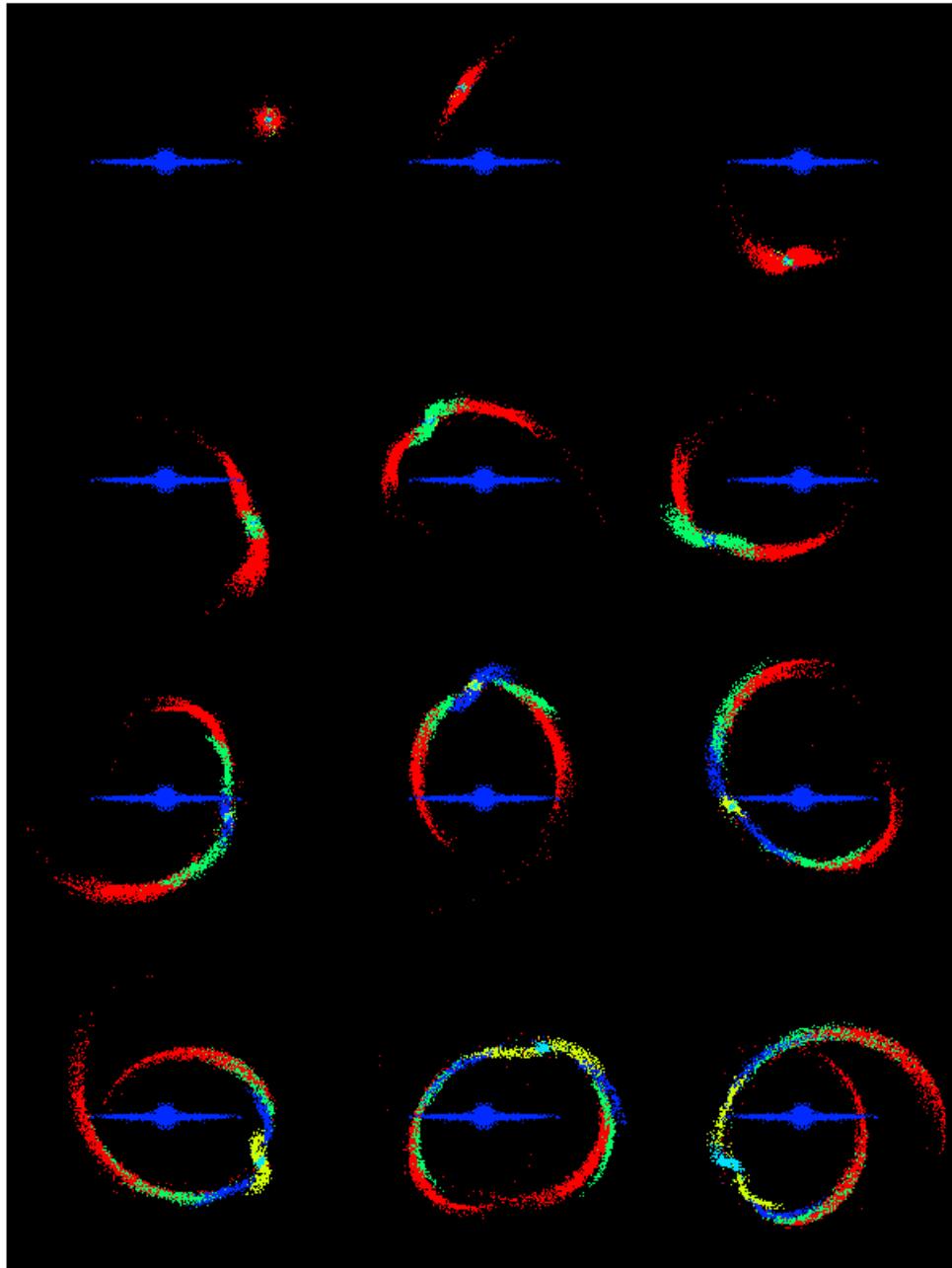
# Milky Way Cannibalism



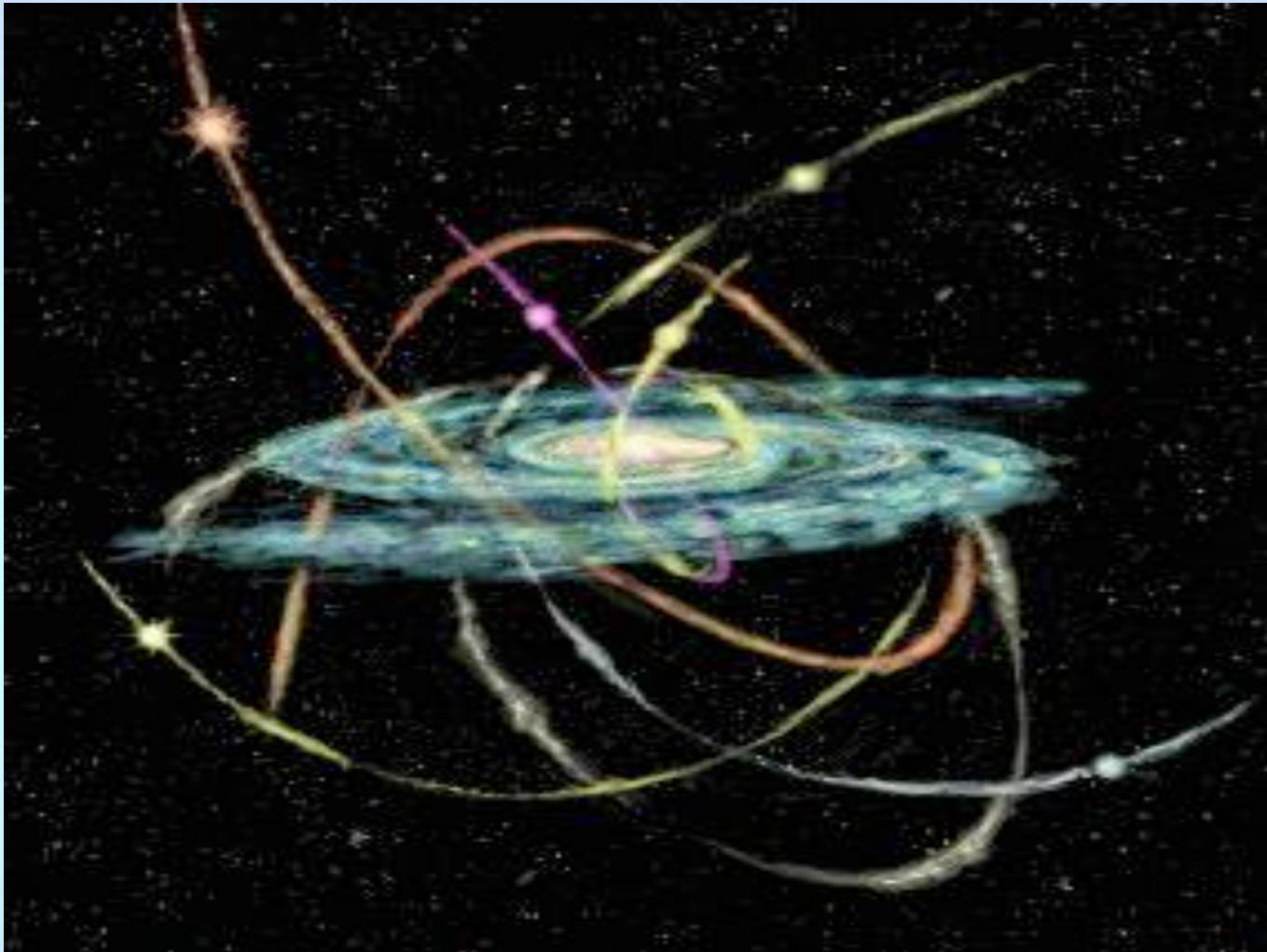
View of the Sagittarius dwarf galaxy

# Milky Way Cannibalism

The Sagittarius  
dwarf galaxy  
from different  
viewpoints



# Milky Way Cannibalism



One can perform galactic archeology using stellar motions and metallicity measurements